AFRL-AFOSR-UK-TR-2017-0018

SATA II - Stochastic Algebraic Topology and Applications 150032

Robert Adler
TECHNION ISRAEL INSTITUTE OF TECHNOLOGY

01/30/2017
Final Report

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/ IOE
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Executive Services, Directorate (0704-0188). Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ORGANIZATION.

1. REPORT DATE *(DD-MM-YYYY)*
15-03-2017

2. REPORT TYPE
Final

3. DATES COVERED *(From - To)*
15 Dec 2014 to 14 Dec 2016

4. TITLE AND SUBTITLE
SATA II - Stochastic Algebraic Topology and Applications

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-15-1-0032

5c. PROGRAM ELEMENT NUMBER
61102F

6. AUTHOR(S)
Robert Adler

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
TECHNION ISRAEL INSTITUTE OF TECHNOLOGY
TECHNION CITY
Haifa, 32000 IL

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
EOARD
Unit 4515
APO AE 09421-4515

10. SPONSOR/MONITOR'S ACRONYM(S)
AFRL/AFOSR IOE

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
AFRL-AFOSR-UK-TR-2017-0018

12. DISTRIBUTION/AVAILABILITY STATEMENT
A DISTRIBUTION UNLIMITED: PB Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT
This report summarizes the previous two years of activity, at the Technion, of what was actually a five year project involving one Israeli and three US principle investigators, and covers those aspects of the project in which the Israeli PI was most active during the past two years. The US part of the project, under Award FA9550-11-1-0216, ran from the period 10/1/2011 09/30/2016. A comprehensive report for this award, which covers the full five year period, and includes the contributions of both the US and Israeli teams, has recently been submitted to AFOSR.

15. SUBJECT TERMS
Network Theory, Sensor Technology, Mathematical Modeling, EOARD

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
Unclassified	Unclassified	Unclassified	SAR	8	PETERSON, JESSE

19b. TELEPHONE NUMBER *(Include area code)*
314 235 6292

EUROPEAN OFFICE OF AEROSPACE RESEARCH AND DEVELOPMENT
AWARD NO. FA9550-15-1-0032

FINAL PERFORMANCE REPORT FOR THE PERIOD
15 December 2014 – 14 December 2016

RESEARCH TITLE
SATA II: Stochastic Algebraic Topology and Applications

Continuation of, and associated with
SATA: Stochastic Algebraic Topology and Applications
FA8655-11-1-3039, 09/1/2011–08/31/2014
FA9550-11-1-0216, 10/1/2011–09/30/2016

PRINCIPAL INVESTIGATOR
Robert J. Adler
Faculty of Electrical Engineering
Technion – Israel Institute of Technology

SUBMISSION DATE
January 23, 2017

Approved for public release; distribution is unlimited.

To avoid doubt, it is hereby emphasized that the Investigator, the Technion Research and Development Foundation Ltd as well as the Technion – Israel Institute of Technology are not and will not be liable for any injury and/or damages and/or expenses and/or loss, of any kind or sort, that is caused or may be caused to the recipient of this report or any third party, directly or incidentally, due to this report or in any relation to it or its implementation.

DISTRIBUTION A. Approved for public release: distribution unlimited.

Table of Contents

Summary

This report summarizes the previous two years of activity, at the Technion, of what was actually a five year project involving one Israeli and three US principle investigators, and covers those aspects of the project in which the Israeli PI was most active during the past two years.

The US part of the project, under Award FA9550-11-1-0216, ran from the period 10/1/2011–09/30/2016. A comprehensive report for this award, which covers the full five year period, and includes the contributions of both the US and Israeli teams, has recently been submitted to AFOSR. The Israeli part of the project also ran over five years, under Award FA8655-11-1-3039 for the period 09/1/2011–08/31/2014, and under FA9550-15-1-0032 for the past two years.

Due to the existence of the full US report, and a final performance report for the first three years (submitted two years ago) of the Israeli part of the project, this current report will not repeat the material of the the earlier ones, but rather give a brief sketch, and detailed listing, of the activity at the Technion over the past two years. As such, it represents a subset of the 5-year US report.

Progress over the past two years has been substantial. The Israeli team continued to work closely with the US PI's, and most of the topics outlined in the original were brought to a successful conclusion. The topics of central interest to the Israeli team were in the areas of the statistics of random functions, random complexes, and random manifolds and embeddings.

In addition, in 2013 significant additional funding for related research at the Technion was obtained from the European Research Council. While the ERC and AFOSR projects are quite distinct, the personnel involved have both common and complimentary interests, and the resultant synergy between the two groups is supportive to both.

Objectives

Recall that the Stochastic Algebraic Topology and Applications (SATA) project aimed to exploit recent advances in the complementary areas of topology and stochastic processes to tackle a wide range of data analytic problems of broad importance. Treating data topologically is crucial in scenarios in which it is important to detect, localize, and perhaps perform an initial classification of objects without attempting to completely characterize them. Adding a stochastic element allows for the almost pervasive situation in which the data itself is imperfectly observed, due to the presence of background noise. As current probabilistic and statistical methodology is ill-suited to detect such qualitative structures, the project aims to develop generic stochastic models whose topological structures are amenable to mathematical analysis, as a first step towards implementation of a broader, more quantitative program. Core topics include random functions on manifolds, random manifolds created by random embeddings, and random manifolds arising in machine learning, along with their theoretical and practical interplay.

Status of Effort

SATA was basically a mathematics based project, and the methods were hard analysis supplemented with, and often motivated by, computation. The core "raw material" for this project is therefore appropriate manpower, which we were successful in drafting at the level of graduate students and postdocs. Some of these are partially funded under SATA, but all benefit from the inter- and intra-disciplinary atmosphere at the Technion and, more globally, with the US side of the project. All of the US collaborators spent time at the Technion during the past two years, and the Israeli PI visited the US. All of these visits have done much to motivate, educate, and stimulate the Technion team.

Accomplishments/New Results

The detailed 5-year report put in by the US side of SATA listed the following areas, and described contributions to each of them.

(1) Statistics of random Functions
(2) Morse theory, critical points, Betti numbers and random complexes
(3) Random manifolds and random embeddings
(4) Other directions
    (a) Complexity bounds
    (b) Multi-scale issues in topological sampling
    (c) Inferential problems related to critical points of common objective functions.
    (d) Application to network modeling
    (e) Miscellaneous applications

Rather than repeat the details if the full report, it is merely noted here that the main contributions of the Israeli team were to areas 1, 2, 3, and 4e. However, as noted in the full report it has "not always been able to separate the work done by the American group from the Israeli team", and contributions of both Israel and US team members to almost all the areas typically followed from, often informal, discussions.

Training of Graduate Students and Postdoctoral Fellows

The entire five year period was very active in terms of graduate students and postdoctoral fellows. Over the past two years Dr. Takashi Owada was partially supported as a postdoc

DISTRIBUTION A. Approved for public release: distribution unlimited.

under SATA, and Dr. Katherine Turner spent a summer at the Technion under SATA support. Dr. Turner had previously been a PhD student under Shmuel Weinberger, one of the US PIs of SATA, and her visit was particularly useful in terms of communication between the groups.

Honors/Awards

(Partial list for Adler, last two years only)

(1) 2015: de Rahm Lecture of the Swiss Doctoral Programme
(2) 2016: British Mathematics Colloquium, Plenary Lecture.

Personnel Supported

(1) Robert Adler. PI. 25%
(2) Dr. Takashi Owada, 2013–2016.
Currently Assistant Professor, Statistics, Purdue.
(3) Dr. Katherine Turner, Summer, 2015.
Currently Postdoctoral Scientist, Laboratory for Topology and Neuroscience. EPFL.

Interactions/Transitions

In addition to the basic research described above, there has been considerable dissemination of the basic ideas of SATA at a number of venues. Major conferences at which Adler spoke over the last two years, or is invited to speak in the near future include:

(1) April, 2015. Annual de Rahm Lecture, EPFL, Lausanne, Switzerland. *Phase Transitions and Random Topology.*
(2) August 2015. Short course on *An Introduction to Random Topology*, Stochastic Geometry Workshop, Poitiers, France.
(3) August 2015. Stochastic Geometry Workshop, Poitiers, France. *Topological Phase Transitions*
(4) September 2015. Heilbronn Annual Conference, Bristol, UK. *Topological Phase Transitions.*
(5) March 2016. British Mathematics Colloquium, Bristol, England. (Plenary) *The Gaussian kinematic formula.*
(6) May 2016. Perspectives on Integral Geometry, Athens, Georgia. *The Gaussian Kinematic Formula: Its Theory and Some of its Applications.*
(7) July 2016. Algebraic Topology: Computation, Methods, and Science (ATMCS), Turin, Italy. *ATMCS and PROBABILITY: An eclectic collection of problems* (Delivered by video due to medical issues which prevented physical travel.)

Further, Adler was involved as co-organiser of the following conferences/workshops:

DISTRIBUTION A. Approved for public release: distribution unlimited.

(1) June, 2015, DyToComp (Dynamics, Topology and Computations), Bedlewo, Poland. Member, Scientific Committee.
(2) June 2015, Extreme Value Analysis, EVA15, Ann Arbor, Michigan. Member, Scientific Committee.
(3) Summer/Fall 2016, Thematic Semester on Probabilistic Methods in Geometry, Topology, and Spectral Theory. Centre de Recherches Mathematiques, Montreal. Member, International Scientific Committee.
(4) June, 2018, DyToComp (Dynamics, Topology and Computations), Bedlewo, Poland. Member, Scientific Committee.

In addition, Adler wrote a series of four columns for the Bulletin of Institute of Mathematical Statistics on the topic of "TOPOS: Topology, Probability and Statistics".

Publications

Scientific publications

Note: This list contains some papers that were either 'in print' in the final report for the the first three years of SATA and have since appeared.

(1) R.J. Adler, O. Bobrowski and S. Weinberger, Crackle: The homology of noise. *Discrete and Computational Geometry*, 52, 2014, 680–704.
(2) O. Bobrowski and R.J. Adler, Distance functions, critical points, and topology for some random complexes. *Homology, Homotopy and Applications*, 16, 2014, 311–344.
(3) D. Yogeshwaran and R.J. Adler, On the topology of random complexes built over stationary point processes. *Annals of Applied Probability* 25, 2015, 3338–3380.
(4) D. Yogeshwaran, E. Subag and R.J. Adler, Random geometric complexes in the thermodynamic regime. *Probability Theory and Related Fields*, 2017. In press. (35 pages).
(5) R.J. Adler and G. Samorodnitsky, Climbing down Gaussian peaks. *Annals of Probability*, 2017. In press. (33 pages)
(6) G. Thoppe, D. Yogeshwaran, and R.J. Adler, On the evolution of topology in dynamic clique complexes. *Advances in Applied Probability*, 48, 2016. In press. (33 pages)
(7) T. Owada and R.J. Adler, Limit theorem for point processes under geometric constraints (and topological crackle) *Annals of Probability*, 2016. In press. (54 pages)
(8) G. Naitzat and R.J. Adler, A central limit theorem for the Euler integral of a Gaussian random field. *Stochastic Processes and its Applications*, 2016. In press. (32 pages)
(9) T. Owada, Functional central limit theorem for subgraph counting processes, *Annals of Probability*, 2016. In press. (35 pages)
(10) R.J. Adler, S.R. Krishnan, J.E. Taylor and S. Weinberger, Convergence of the reach for a sequence of Gaussian-embedded manifolds. Submitted for publication. (50 pages)
(11) S.R. Krishnan, J.E. Taylor and R.J. Adler, The intrinsic geometry of some random manifolds. *Electronic Communications in Probability*, (2017) In press. (12 pages)

DISTRIBUTION A. Approved for public release: distribution unlimited.

Popular articles

(1) R.J. Adler, *TOPOS*: Applied topologists do it with persistence, *Bulletin IMS*, 43–6, 2014, 10–11.
(2) R.J. Adler, *TOPOS*: Pinsky was wrong, Euler was right. *Bulletin IMS*, 43–8, 2014, 6–7.
(3) R.J. Adler, *TOPOS*: Let's not make the same mistake twice. *Bulletin IMS*, 44–2, 2015, 4–5.

DISTRIBUTION A. Approved for public release: distribution unlimited.